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Transient motion of an anisotropic elastic bimaterial due to a line source

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Abstract

The transient motion of an anisotropic elastic bimaterial due to a line force or a line dislocation is studied. The bimaterial is assumed to be at rest and stress-free for $t < 0$. The line source is applied at $t = 0$ and maintained for $t > 0$. A formulation which is an extension to Stroh's formalism for anisotropic elastostatics is employed. The general solution is expressed in terms of the eigenvalues and eigenvectors of a related eigenvalue problem. The method is used to obtain the analytic solutions without the need of performing integral transforms. Numerical examples of the GaAs bimaterial due to a line force or dislocation are presented for illustration.

Keywords: Transient motion; Anisotropic elastic material; Bimaterial; Line source

1. Introduction

Wave propagation in layered elastic media has been a subject of interest in the field of geophysics, acoustics and nondestructive testing. Analysis of the elastodynamic problem is complicated due to the fact that as the propagating wave is interrupted by the interfaces, reflection and transmission waves occur and interfacial waves may also arise. The degree of complexity of the interactions depends on the mechanical properties of the individual layer, number and nature of the interfacial conditions and loading conditions, among other factors. Here we consider the dynamic response of a bimaterial composed of two dissimilar elastic half-spaces of general anisotropy induced by a line force or a line dislocation. The problem serves as a basis for further studies of anisotropic layered systems.

Ma and Huang (1996) considered the problem for an isotropic bimaterial loaded by a line force. The problem is solved by application of Laplace transform method. The inverse transforms are evaluated by means of Cagniard's method. Every and Briggs (1998) presented algorithms based on Fourier transform for calculating the time domain displacement response of fluid-loaded anisotropic half-spaces to impulsive line and point forces at their interface. Wu (2003) have used an extended Stroh's formalism to derive a closed-form solution

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for a suddenly applied interfacial line force or dislocation in an anisotropic bimaterial. In this formulation the solution is expressed in terms of the eigenvalues and eigenvectors of a six-dimensional matrix, which is a function of the material constants, time and position. A major advantage of the formulation is that no integral transforms are required. The fact greatly facilitates derivations of explicit solutions. Recently, Wu and Chen (2006) have further generalized the formulation to treat the problem of a dynamic buried source in a semi-infinite medium. In this paper the generalized formalism proposed by Wu and Chen is used.

The plan of the paper is as follows. In Section 2 the formulation employed is introduced. The problem of a buried line source in an anisotropic bimaterial is studied in Section 3. Numerical examples are given in Section 4. Some conclusions are finally given.

2. Formulation

For two-dimensional deformation in which the Cartesian components of the stress σ_{ij} and the displacement $u_i, i,j = 1,2,3$, are independent of x_3, the equations of motion are

$$\mathbf{t}_{1,1} + \mathbf{t}_{2,2} = \rho\ddot{\mathbf{u}}, \tag{1}$$

where $\mathbf{t}_1 = (\sigma_{11}, \sigma_{21}, \sigma_{31})^{\mathrm{T}}$, $\mathbf{t}_2 = (\sigma_{12}, \sigma_{22}, \sigma_{32})^{\mathrm{T}}$, $\ddot{\mathbf{u}}$ is the acceleration, ρ is the density, a subscript comma denotes partial differentiation with respect to coordinates and overhead dot designates derivative with respect to time t. The stress–strain laws are

$$\mathbf{t}_1 = \mathbf{Q}\mathbf{u}_{,1} + \mathbf{S}\mathbf{u}_{,2}, \tag{2}$$

$$\mathbf{t}_2 = \mathbf{S}^{\mathrm{T}}\mathbf{u}_{,1} + \mathbf{W}\mathbf{u}_{,2}, \tag{3}$$

where the matrices $\mathbf{Q}$, $\mathbf{S}$, and $\mathbf{W}$ are related to elastic constants C_{ijks} by

$$Q_{ik} = C_{i1k1}, \quad S_{ik} = C_{i1k2}, \quad W_{ik} = C_{i2k2}.$$

The equations of motion expressed in terms of displacements are obtained by substituting Eqs. (2) and (3) into Eq. (1) as

$$\mathbf{Q}\mathbf{u}_{,11} + (\mathbf{S} + \mathbf{S}^{\mathrm{T}})\mathbf{u}_{,12} + \mathbf{W}\mathbf{u}_{,22} = \rho\ddot{\mathbf{u}}. \tag{4}$$

Let the displacement be assumed as

$$\mathbf{u}(x_1, x_2, t) = \mathbf{u}(w) \tag{5}$$

with the variable $w(x_1, x_2, t)$ implicitly defined by

$$wt - x_1 - p(w)x_2 - q(w) = 0, \tag{6}$$

where $p(w)$ and $q(w)$ are functions of w.

With Eqs. (5) and (6), Eq. (4) becomes (Wu and Chen, 2006)

$$\frac{\partial w}{\partial x_1}\frac{\partial}{\partial w}\left\{[\mathbf{Q} - \rho w^2\mathbf{I} + p(w)(\mathbf{S} + \mathbf{S}^{\mathrm{T}}) + p(w)^2\mathbf{W}]\frac{\partial w}{\partial x_1}\mathbf{u}'(w)\right\} = \mathbf{0}, \tag{7}$$

where $\mathbf{I}$ is the identity matrix and

$$\frac{\partial w}{\partial x_1} = \frac{1}{t - p'(w)x_2 - q'(w)}. \tag{8}$$

Let $\mathbf{u}'(w)$ be expressed as

$$\mathbf{u}'(w) = f(w)\mathbf{a}(w), \tag{9}$$

where $f(w)$ is an arbitrary scalar function of w. It follows that $\mathbf{u}(w)$ is a solution of Eq. (4) if

$$\mathbf{D}(p, w)\mathbf{a}(w) = \mathbf{0}, \tag{10}$$

where $\mathbf{D}(p, w)$ is given by

$$\mathbf{D}(p, w) = \mathbf{Q} + p(\mathbf{S} + \mathbf{S}^{\mathrm{T}}) + p^2\mathbf{W} - \rho w^2\mathbf{I}. \tag{11}$$

For non-trivial solutions of $\mathbf{a}(w)$ we must have

$$|\mathbf{D}(p, w)| = 0, \tag{12}$$

where $|\mathbf{D}|$ is the determinant of $\mathbf{D}$.

Eq. (12) provides six eigenvalues of p as a function of w, denoted by $p_k(w)$, $k = 1, 2, \ldots, 6$. The function $p_k(w)$ is single-valued if w is allowed to range over the six sheets $\sum^k$ of its Riemann surface, taking the values $p_k(w)$ on $\sum^k$ (Willis, 1973). If w is real and $|w|$ is sufficiently large, there are six real roots $p_k(w)$ such that (Wu, 2000)

$$\frac{\mathrm{d}w}{\mathrm{d}p} = \xi_2, \tag{13}$$

where ξ_2 is the x_2 component of the ray velocity. Three of these roots characterized by $\mathrm{d}w/\mathrm{d}p > 0$ are associated with the rays propagating in the direction of positive x_2 direction and the others by $\mathrm{d}w/\mathrm{d}p < 0$ with the rays propagating in the direction of negative x_2 direction. The three of the former type will be assigned to the Riemann surfaces $\sum^k (k = 1, 2, 3)$ and the three of the latter type to $\sum^k (k = 4, 5, 6)$. The sheets are connected across appropriate lines joining the branch points of $p_k(w)$, which are located on the real axis in the complex w-plane and are determined by $\mathrm{d}w/\mathrm{d}p = 0$. It can be shown that $p_k(w)$ has positive imaginary part in the upper half of $\sum^k (k = 1, 2, 3)$ and negative imaginary part in the upper plane of $\sum^k (k = 4, 5, 6)$.

The general solution of Eq. (4) may be represented as

$$\mathbf{u}(x_1, x_2, t)_{,1} = 2\,\mathrm{Re}\left\{\sum_k f_k(w_k)\frac{\partial w_k}{\partial x_1}\mathbf{a}_k(w_k)\right\}, \tag{14}$$

$$\mathbf{u}(x_1, x_2, t)_{,2} = 2\,\mathrm{Re}\left\{\sum_k p_k(w_k) f_k(w_k)\frac{\partial w_k}{\partial x_1}\mathbf{a}_k(w_k)\right\}, \tag{15}$$

$$\dot{\mathbf{u}}(x_1, x_2, t)_{,1} = -2\,\mathrm{Re}\left\{\sum_k w_k f_k(w_k)\frac{\partial w_k}{\partial x_1}\mathbf{a}_k(w_k)\right\}, \tag{16}$$

where $f_k(w_k)$ is an arbitrary function of w_k and $k = 1, 2, 3$ or $4, 5, 6$. The choice of the range of k depends on whether up-going rays or down-going rays are considered.

By substituting Eqs. (14) and (15) into Eqs. (2) and (3), the general solutions of the stress vectors $\mathbf{t}_1$ and $\mathbf{t}_2$ can be expressed as

$$\mathbf{t}_1(x_1, x_2, t) = 2\,\mathrm{Re}\left\{\sum_k f_k(w_k)\left[\rho w_k^2\frac{\partial w_k}{\partial x_1}\mathbf{a}_k(w_k) - p_k(w_k)\frac{\partial w_k}{\partial x_1}\mathbf{b}_k(w_k)\right]\right\}, \tag{17}$$

$$\mathbf{t}_2(x_1, x_2, t) = 2\,\mathrm{Re}\left\{\sum_k f_k(w_k)\frac{\partial w_k}{\partial x_1}\mathbf{b}_k(w_k)\right\}, \tag{18}$$

where

$$\mathbf{b}_k(w) = (\mathbf{S}^{\mathrm{T}} + p_k(w)\mathbf{W})\mathbf{a}_k(w) = -\frac{1}{p}(\mathbf{Q} - \rho w^2\mathbf{I} + p_k(w)\mathbf{S})\mathbf{a}_k(w). \tag{19}$$

The second identity in Eq. (19) follows from Eq. (10). Eq. (19) can be cast into the following six-dimensional eigenvalue problem

$$\mathbf{N}(w)\boldsymbol{\xi}(w) = p(w)\boldsymbol{\xi}(w), \tag{20}$$

where

$$\mathbf{N}(w) = \begin{pmatrix} \mathbf{N}_1 & \mathbf{N}_2 \\ \mathbf{N}_3(w) & \mathbf{N}_1^{\mathrm{T}} \end{pmatrix}, \quad \boldsymbol{\xi}(w) = \begin{pmatrix} \mathbf{a}(w) \\ \mathbf{b}(w) \end{pmatrix},$$

$$\mathbf{N}_1 = -\mathbf{W}^{-1}\mathbf{S}^{\mathrm{T}}, \quad \mathbf{N}_2 = \mathbf{W}^{-1}, \quad \mathbf{N}_3(w) = \mathbf{S}\mathbf{W}^{-1}\mathbf{S}^{\mathrm{T}} - \mathbf{Q} + \rho w^2\mathbf{I}.$$

Eq. (20) is in the same form as that in Stroh's formalism for steady state motion (Stroh, 1962). Let

$$\mathbf{A}(w) = [\mathbf{a}_1(w) \quad \mathbf{a}_2(w) \quad \mathbf{a}_3(w)], \quad \mathbf{B}(w) = [\mathbf{b}_1(w) \quad \mathbf{b}_2(w) \quad \mathbf{b}_3(w)], \tag{21}$$

$$\widehat{\mathbf{A}}(w) = [\mathbf{a}_4(w) \quad \mathbf{a}_5(w) \quad \mathbf{a}_6(w)], \quad \widehat{\mathbf{B}}(w) = [\mathbf{b}_4(w) \quad \mathbf{b}_5(w) \quad \mathbf{b}_6(w)]. \tag{22}$$

These matrices satisfy the closure relations (Ting, 1996, p. 445)

$$\mathbf{A}(w)\mathbf{A}^{\mathrm{T}}(w) + \widehat{\mathbf{A}}(w)\widehat{\mathbf{A}}^{\mathrm{T}}(w) = \mathbf{0} = \mathbf{B}(w)\mathbf{B}^{\mathrm{T}}(w) + \widehat{\mathbf{B}}(w)\widehat{\mathbf{B}}^{\mathrm{T}}(w), \tag{23}$$

$$\mathbf{A}(w)\mathbf{B}^{\mathrm{T}}(w) + \widehat{\mathbf{A}}(w)\widehat{\mathbf{B}}^{\mathrm{T}}(w) = \mathbf{I} = \mathbf{B}(w)\mathbf{A}^{\mathrm{T}}(w) + \widehat{\mathbf{B}}(w)\widehat{\mathbf{A}}^{\mathrm{T}}(w), \tag{24}$$

if the eigenvectors $\boldsymbol{\xi}_\alpha(w)$, $\alpha = 1,2,\ldots,6$, are normalized such that

$$\mathbf{a}_k^{\mathrm{T}}(w)\mathbf{b}_j(w) + \mathbf{b}_k^{\mathrm{T}}(w)\mathbf{a}_j(w) = \delta_{kj}, \tag{25}$$

where δ_{kj} is the Kronecker delta.

3. A line force and dislocation in a bimaterial

Consider a bimaterial consisting of two dissimilar elastic half-spaces perfectly bonded together. Let the half-space $x_2 \geqslant 0$ be occupied by material 1 and the half-space $x_2 \leqslant 0$ be occupied by material 2. The bimaterial is initially stress-free and is subjected to a line force $H(t)\mathbf{F}$ and a dislocation of Burgers vector $H(t)\boldsymbol{\beta}$ at $x_1 = 0$ and $x_2 = h > 0$,$H(t)$ being a Heaviside step function. Fig. 1 summarizes the configuration of this problem. The continuity conditions at the interface $x_2 = 0$ are given by

$$\mathbf{u}_{,1}(x_1, 0^+, t) = \mathbf{u}^*_{,1}(x_1, 0^-, t), \tag{26}$$

$$\mathbf{t}_2(x_1, 0^+, t) = \mathbf{t}^*_2(x_1, 0^-, t), \tag{27}$$

where the superscript "*" denotes quantities referred to material 2. Let $\mathbf{u}_{,1}$ ant $\mathbf{t}_2$ in the upper half-space be expressed as

$$\mathbf{u}_{,1} = \mathbf{u}^{(0)}_{,1} + \mathbf{u}^{(1)}_{,1}, \tag{28}$$

$$\mathbf{t}_2 = \mathbf{t}^{(0)}_2 + \mathbf{t}^{(1)}_2, \tag{29}$$

where $\mathbf{u}^{(0)}_{,1}$ and $\mathbf{t}^{(0)}_2$ are, respectively, the displacement gradient and the stress vector due to the sources in an infinite medium, $\mathbf{u}^{(1)}_{,1}$ and $\mathbf{t}^{(1)}_2$ are those due to the reflected waves from the interface.

The solution for the line force in an infinite medium has been obtained by Wu (2000) and that for the line dislocation may be derived similarly. The result is

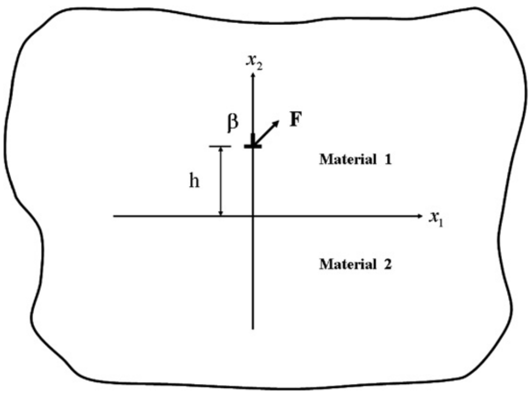

Fig. 1. A line force **F** and a line dislocation with Burgers vector **β** in an infinite bimaterial.

$$\dot{\mathbf{u}}^{(0)} = \frac{1}{\pi}\,\mathrm{Im}\left\{\sum_{k=4}^{6} c_k(w_k)\frac{\partial w_k}{\partial x_1}\mathbf{a}_k(w_k)H(t-\hat{t}_k)\right\}, \tag{30}$$

$$\mathbf{u}^{(0)}_{,1} = -\frac{1}{\pi}\,\mathrm{Im}\left\{\sum_{k=4}^{6} \frac{c_k(w_k)}{w_k}\frac{\partial w_k}{\partial x_1}\mathbf{a}_k(w_k)H(t-\hat{t}_k)\right\}, \tag{31}$$

$$\mathbf{t}^{(0)}_{2} = -\frac{1}{\pi}\,\mathrm{Im}\left\{\sum_{k=4}^{6} \frac{c_k(w_k)}{w_k}\frac{\partial w_k}{\partial x_1}\mathbf{b}_k(w_k)H(t-\hat{t}_k)\right\}, \tag{32}$$

where $c_k(w_k) = \mathbf{a}_k^{\mathrm{T}}(w_k)\mathbf{F} + \mathbf{b}_k^{\mathrm{T}}(w_k)\boldsymbol{\beta}$, w_k is determined by

$$wt = x_1 + p_k(w)(x_2 - h) \tag{33}$$

and $\hat{t}_k$ is the arrival time of the bulk wave corresponding to p_k given by

$$\hat{t}_k = \left|\frac{x_2 - h}{\xi_2^{(k)}}\right|. \tag{34}$$

Here $\xi_2^{(k)}$ is the velocity of the ray radiating from $(0, h)$ to (x_1, x_2). The reason for including the Heaviside functions is as follows. Eqs. (30)–(32) imply that for the particle velocity and stress to be nonzero, w_k and $p_k(w_k)$ must be complex. If $p_k(w_k)$ is real, for a fixed point (x_1, x_2), Eq. (33) may be considered as a function of t in terms of real w_k. The maximum time $t_{\max}$ that can result from this function is obtained by

$$\frac{\mathrm{d}t}{\mathrm{d}w} = 0 \quad \text{or } t_{\max} = p_k'(w)(x_2 - h).$$

From Eq. (13), it may be seen that $t_{\max}$ is in fact $\hat{t}_k$. Thus $p_k(w_k)$ is complex only if $t > \hat{t}_k$. Physically this means that the disturbances at a point occur only when the bulk wave reaches the point.

Since up-going waves are generated in material 1 and down-going waves are in material 2, the expressions for $\mathbf{u}^{(1)}_{,1}$ and $\mathbf{t}^{(1)}_{2}$ in material 1 are given by

$$\mathbf{u}^{(1)}_{,1}(x_1, x_2, t) = -\frac{1}{\pi}\sum_{k=4}^{6}\sum_{j=1}^{3}\mathrm{Im}\left\{R_{kj}(w_{kj})\frac{c_k(w_{kj})}{w_{kj}}\frac{\partial w_{kj}}{\partial x_1}\mathbf{a}_j(w_{kj})H(t-\hat{t}_{kj})\right\}, \tag{35}$$

$$\mathbf{t}^{(1)}_{2}(x_1, x_2, t) = -\frac{1}{\pi}\sum_{k=4}^{6}\sum_{j=1}^{3}\mathrm{Im}\left\{R_{kj}(w_{kj})\frac{c_k(w_{kj})}{w_{kj}}\frac{\partial w_{kj}}{\partial x_1}\mathbf{b}_j(w_{kj})H(t-\hat{t}_{kj})\right\}, \tag{36}$$

where R_{kj} may be regarded as the reflection coefficients,

$$w_{kj}t = x_1 + p_j(w_{kj})x_2 - p_k(w_{kj})h \tag{37}$$

and $\hat{t}_{kj}$ is the arrival time of the reflected wave associated with p_j due to the incident wave related to p_k. Following the same discussion as for $\hat{t}_k$, the expression for $\hat{t}_{kj}$ is given by

$$\hat{t}_{kj} = \left|\frac{x_2}{\xi_2^{(j)}}\right| + \left|\frac{h}{\xi_2^{(k)}}\right|. \tag{38}$$

Those for material 2 are

$$\mathbf{u}^{*}_{,1}(x_1, x_2, t) = -\frac{1}{\pi}\sum_{k=4}^{6}\sum_{j=4}^{6}\mathrm{Im}\left\{T_{kj}(w^{*}_{kj})\frac{c_k(w^{*}_{kj})}{w^{*}_{kj}}\frac{\partial w^{*}_{kj}}{\partial x_1}\mathbf{a}^{*}_j(w^{*}_{kj})H(t-\hat{t}^{*}_{kj})\right\}, \tag{39}$$

$$\mathbf{t}^{*}_{2}(x_1, x_2, t) = -\frac{1}{\pi}\sum_{k=4}^{6}\sum_{j=4}^{6}\mathrm{Im}\left\{T_{kj}(w^{*}_{kj})\frac{c_k(w^{*}_{kj})}{w^{*}_{kj}}\frac{\partial w^{*}_{kj}}{\partial x_1}\mathbf{b}^{*}_j(w^{*}_{kj})H(t-\hat{t}^{*}_{kj})\right\}, \tag{40}$$

where T_{kj} may be regarded as the transmission coefficients,

$$w^{*}_{kj}t = x_1 + p^{*}_j(w^{*}_{kj})x_2 - p_k(w^{*}_{kj})h \tag{41}$$

and $\hat{t}^*_{kj}$ is the arrival time of the transmitted wave associated with p^*_j due to the incident wave related to p_k. The expression for $\hat{t}^*_{kj}$ is given by

$$\hat{t}^*_{kj} = \left|\frac{x_2}{\xi_2^{*(j)}}\right| + \left|\frac{h}{\xi_2^{(k)}}\right|. \tag{42}$$

Note that at the interface $x_2 = 0$,

$$w_{kj}(x_1, t) = w^*_{kj}(x_1, t) = w_k(x_1, t). \tag{43}$$

Eqs. (35) and (36) become

$$\mathbf{u}^{(1)}_{,1}(x_1, t) = -\frac{1}{\pi}\sum_{k=4}^{6} \mathrm{Im}\left\{\frac{c_k(w_k)}{w_k}\frac{\partial w_k}{\partial x_1}\mathbf{A}(w_k)\mathbf{R}_k(w_k)H(t-\hat{t}_k)\right\}. \tag{44}$$

$$\mathbf{t}^{(1)}_{2}(x_1, t) = -\frac{1}{\pi}\sum_{k=4}^{6} \mathrm{Im}\left\{\frac{c_k(w_k)}{w_k}\frac{\partial w_k}{\partial x_1}\mathbf{B}(w_k)\mathbf{R}_k(w_k)H(t-\hat{t}_k)\right\}, \tag{45}$$

where

$$\mathbf{R}_k(w_k) = [R_{k1}(w_k)\ R_{k2}(w_k)\ R_{k3}(w_k)]^{\mathrm{T}},$$

$$\mathbf{A}(w_k) = [\mathbf{a}_1(w_k)\ \mathbf{a}_2(w_k)\ \mathbf{a}_3(w_k)], \tag{46}$$

$$\mathbf{B}(w_k) = [\mathbf{b}_1(w_k)\ \mathbf{b}_2(w_k)\ \mathbf{b}_3(w_k)] \tag{47}$$

and Eqs. (39) and (40) become

$$\mathbf{u}^{*}_{,1}(x_1, t) = -\frac{1}{\pi}\sum_{k=4}^{6} \mathrm{Im}\left\{\frac{c_k(w_k)}{w_k}\frac{\partial w_k}{\partial x_1}\widehat{\mathbf{A}}^*(w_k)\widehat{\mathbf{T}}_k(w_k)H(t-\hat{t}_k)\right\}, \tag{48}$$

$$\mathbf{t}^{*}_{2}(x_1, t) = -\frac{1}{\pi}\sum_{k=4}^{6} \mathrm{Im}\left\{\frac{c_k(w_k)}{w_k}\frac{\partial w_k}{\partial x_1}\widehat{\mathbf{B}}^*(w_k)\widehat{\mathbf{T}}_k(w_k)H(t-\hat{t}_k)\right\}, \tag{49}$$

where

$$\widehat{\mathbf{T}}_k(w_k) = [T_{k4}(w_k)\ T_{k5}(w_k)\ T_{k6}(w_k)]^{\mathrm{T}},$$

$$\widehat{\mathbf{A}}^*(w_k) = [\mathbf{a}^*_4(w_k)\ \mathbf{a}^*_5(w_k)\ \mathbf{a}^*_6(w_k)], \tag{50}$$

$$\widehat{\mathbf{B}}^*(w_k) = [\mathbf{b}^*_4(w_k)\ \mathbf{b}^*_5(w_k)\ \mathbf{b}^*_6(w_k)]. \tag{51}$$

The continuity conditions of Eqs. (26) and (27) then lead to

$$\mathbf{A}(w_k)\mathbf{R}_k(w_k) - \widehat{\mathbf{A}}^*(w_k)\widehat{\mathbf{T}}_k(w_k) = -\mathbf{a}_k(w_k), \tag{52}$$

$$\mathbf{B}(w_k)\mathbf{R}_k(w_k) - \widehat{\mathbf{B}}^*(w_k)\widehat{\mathbf{T}}_k(w_k) = -\mathbf{b}_k(w_k), \tag{53}$$

$k = 4, 5, 6$. Solving Eqs. (52) and (53) yields

$$\mathbf{R}_k(w_k) = \mathbf{A}^{-1}(w_k)\mathbf{M}^{-1}(w_k)[-\widehat{\mathbf{M}}_2(w_k)\mathbf{a}_k(w_k) + \mathrm{i}\mathbf{b}_k(w_k)], \tag{54}$$

$$\widehat{\mathbf{T}}(w_k) = [\widehat{\mathbf{A}}^*(w_k)]^{-1}\mathbf{M}^{-1}(w_k)[\mathbf{M}_1(w_k)\mathbf{a}_k(w_k) + \mathrm{i}\mathbf{b}_k(w_k)], \tag{55}$$

where $\mathbf{M}_1(w_k) = -\mathrm{i}\mathbf{B}(w_k)\mathbf{A}^{-1}(w_k)$, $\widehat{\mathbf{M}}_2(w_t) = \mathrm{i}\widehat{\mathbf{B}}^*(w_t)[\widehat{\mathbf{A}}^*(w_t)]^{-1}$ are the impedance tensors (Lothe and Barnett, 1976) of materials 1 and 2, respectively, and $\mathbf{M}(w_k)$ is given by

$$\mathbf{M}(w_k) = \mathbf{M}_1(w_k) + \widehat{\mathbf{M}}_2(w_k). \tag{56}$$

Note that the interfacial Stoneley wave speed v_s is determined by $|\mathbf{M}(v_s)| = 0$ (Chadwick and Currie, 1974). The functions $R_{kj}(w_{kj})$ and $T_{kj}(w^*_{kj})$ can be obtained from $\mathbf{R}_k$ and $\widehat{\mathbf{T}}_k$, respectively, as

$$R_{kj}(w_{kj}) = \mathbf{e}^{\mathrm{T}}_j\mathbf{R}_k(w_{kj}),\quad j = 1, 2, 3;\quad T_{kj}(w^*_{kj}) = \mathbf{e}^{\mathrm{T}}_{j-3}\widehat{\mathbf{T}}_k(w^*_{kj}),\quad j = 4, 5, 6,\ \ k = 4, 5, 6. \tag{57}$$

Three special cases: (a) $t \to \infty$, (b) $h \to 0$, and (c) $x_2 = 0$, are discussed as follows:

(a) For $t \to \infty$, $w_{kj} \to 0$ and $w^*_{kj} \to 0$ such that

$$\frac{\partial w_{kj}}{\partial x_1} \to \frac{1}{t}, \quad w_{kj}t \to z_j - p_k(0)h, \quad \frac{\partial w^*_{kj}}{\partial x_1} \to \frac{1}{t} \quad \text{and} \quad w^*_{kj}t \to z^*_j - p_k(0)h, \tag{58}$$

where $z_j = x_1 + p_j(0)x_2$ and $z^*_j = x_1 + p^*_j(0)x_2$. Eqs. (32), (36) and (40) becomes

$$\mathbf{t}_2^{(0)}(x_1, x_2, t) = \frac{1}{\pi}\,\mathrm{Im}\left\{\mathbf{B}(0)\left\langle\frac{1}{z_* - p_*(0)h}\right\rangle\mathbf{c}^\infty(0)\right\}, \tag{59}$$

$$\mathbf{t}_2^{(1)}(x_1, x_2, t) = \frac{1}{\pi}\sum_{k=1}^{3}\mathrm{Im}\left\{\mathbf{B}(0)\left\langle\frac{1}{z_* - \bar{p}_k(0)h}\right\rangle\mathbf{A}^{-1}(0)^{-1}\mathbf{M}^{-1}(0)[\widehat{\mathbf{M}}_2(0) - \overline{\mathbf{M}}_1(0)]\overline{\mathbf{A}}(0)\mathbf{I}_k\bar{\mathbf{c}}^\infty(0)\right\}, \tag{60}$$

$$\mathbf{t}_2^{*}(x_1, x_2, t) = \frac{1}{\pi}\sum_{k=1}^{3}\mathrm{Im}\left\{\mathbf{B}^*(0)\left\langle\frac{1}{z^*_* - p_k(0)h}\right\rangle[\mathbf{A}^*(0)]^{-1}\overline{\mathbf{M}}^{-1}(0)[\mathbf{M}_1(0) + \overline{\mathbf{M}}_1(0)]\mathbf{A}(0)\mathbf{I}_k\mathbf{c}^\infty(0)\right\}, \tag{61}$$

where $\mathbf{I}_k = \mathbf{e}_k\mathbf{e}_k^{\mathrm{T}}$ and $\mathbf{e}_k$ is the unit vector in the x_k-direction,

$$\mathbf{c}^\infty(0) = [c_1(0) \quad c_2(0) \quad c_3(0)]^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}}(0)\mathbf{F} + \mathbf{B}^{\mathrm{T}}(0)\boldsymbol{\beta},$$

$\left\langle\frac{1}{z_* - p_*(0)h}\right\rangle$ and $\left\langle\frac{1}{z^*_* - p_k(0)h}\right\rangle$ are the diagonal matrices given by

$$\left\langle\frac{1}{z_* - p_*(0)h}\right\rangle = \mathrm{diag}\left(\frac{1}{z_1 - p_1(0)h}, \frac{1}{z_2 - p_2(0)h}, \frac{1}{z_3 - p_3(0)h}\right),$$
$$\left\langle\frac{1}{z^*_* - p_k(0)h}\right\rangle = \mathrm{diag}\left(\frac{1}{z^*_1 - p_k(0)h}, \frac{1}{z^*_2 - p_k(0)h}, \frac{1}{z^*_3 - p_k(0)h}\right).$$

In Eqs. (59)–(61), the following replacements have been made

$$p_{k+3}(0) = \bar{p}_k(0), \quad p^*_{k+3}(0) = \bar{p}^*_k(0), \quad k = 1, 2, 3,$$
$$\widehat{\mathbf{A}}(0) = \overline{\mathbf{A}}(0), \quad \widehat{\mathbf{B}}(0) = \overline{\mathbf{B}}(0), \quad \widehat{\mathbf{A}}^*(0) = \overline{\mathbf{A}}^*(0), \quad \widehat{\mathbf{B}}^*(0) = \overline{\mathbf{B}}^*(0).$$

Eqs. (59)–(61) recover the static result (Ting, 1996).

(b) When $h \to 0$, $w_{kj} = w_j$, $w^*_{kj} = w^*_j$, where w_j, w^*_j, $\frac{\partial w_j}{\partial x_1}$ and $\frac{\partial w^*_j}{\partial x_1}$ are given by

$$w_j t = x_1 + p_j(w_j)x_2, \quad w^*_j t = x_1 + p^*_j(w^*_j)x_2,$$
$$\frac{\partial w_j}{\partial x_1} = \frac{1}{t - p'_j(w_j)x_2} \quad \text{and} \quad \frac{\partial w^*_j}{\partial x_1} = \frac{1}{t - p^{*\prime}_j(w^*_j)x_2}.$$

The velocity $\dot{\mathbf{u}}^{(1)}$ can be written as

$$\dot{\mathbf{u}}^{(1)}(x_1, x_2, t) = \frac{1}{\pi}\,\mathrm{Im}\left\{\sum_{j=1}^{3}\frac{\partial w_j}{\partial x_1}\mathbf{A}(w_j)\mathbf{I}_j\left\{\mathbf{A}^{-1}(w_j)\mathbf{M}^{-1}(w_j)[-\widehat{\mathbf{M}}_2(w_j) + \widehat{\mathbf{A}}(w_j) + \mathrm{i}\widehat{\mathbf{B}}(w_j)]\hat{\mathbf{c}}(w_j)\right\}\right\}, \tag{62}$$

where

$$\hat{\mathbf{c}}(w_j) = \widehat{\mathbf{A}}^{\mathrm{T}}(w_j)\mathbf{F} + \widehat{\mathbf{B}}^{\mathrm{T}}(w_j)\boldsymbol{\beta}.$$

On the other hand, Eq. (30) with $h \to 0$ may be expressed as

$$\dot{\mathbf{u}}(x_1, x_2, t) = -\frac{1}{\pi}\,\mathrm{Im}\left\{\sum_{j=1}^{3}\frac{\partial w_j}{\partial x_1}\mathbf{A}(w_j)\mathbf{I}_j\{\mathbf{A}^{-1}(w_j)\mathbf{M}^{-1}(w_j)\mathbf{M}(w_j)\mathbf{A}(w_j)\mathbf{c}(w_j)\}H(t - \hat{t}_j)\right\}, \tag{63}$$

where $\mathbf{c}(w_j) = \mathbf{A}^{\mathrm{T}}(w_j)\mathbf{F} + \mathbf{B}^{\mathrm{T}}(w_j)\boldsymbol{\beta}$.

The total velocity $\dot{\mathbf{u}}$ for material 1 obtained by adding Eqs. (62) and (63) is given by

$$\dot{\mathbf{u}}(x_1,x_2,t) = -\frac{1}{\pi}\,\mathrm{Im}\left\{\sum_{j=1}^{3}\frac{\partial w_j}{\partial x_1}\mathbf{A}(w_j)\mathbf{I}_j\mathbf{A}^{-1}(w_j)\mathbf{M}^{-1}(w_j)[\widehat{\mathbf{M}}_2(w_j)\boldsymbol{\beta} - i\hat{\mathbf{F}}]H(t-\hat{t}_j)\right\}. \tag{64}$$

Similarly the velocity $\dot{\mathbf{u}}^*$ for material 2 can be simplified as

$$\dot{\mathbf{u}}^*(x_1,x_2,t) = \frac{1}{\pi}\,\mathrm{Im}\left\{\sum_{j=4}^{6}\frac{\partial w_j^*}{\partial x_1}\hat{\mathbf{A}}^*(w_j^*)\mathbf{I}_{j-3}[\widehat{\mathbf{A}}^*(w_j^*)]^{-1}\mathbf{M}^{-1}(w_j^*)[\mathbf{M}_1(w_j^*)\boldsymbol{\beta} + i\hat{\mathbf{F}}]H(t-\hat{t}_j^*)\right\}. \tag{65}$$

In deriving Eqs. (64) and (65), Eqs. (23) and (24) have been used. Eqs. (64) and (65) are identical with the result derived by Wu (2003).

(c) At $x_2 = 0$, $w_{kj} = w_{kj}^* = w_k$, therefore, the velocity $\dot{\mathbf{u}}$ at the interface may be simplified as

$$\dot{\mathbf{u}}(x_1,t) = \frac{1}{\pi}\sum_{k=4}^{6}\mathrm{Re}\left\{\frac{\partial w_k}{\partial x_1}\mathbf{M}^{-1}(w_k)[\widehat{\mathbf{A}}^{\mathrm{T}}(w_k)]^{-1}\mathbf{I}_{k-3}[\widehat{\mathbf{A}}^{\mathrm{T}}(w_k)\mathbf{F} + \widehat{\mathbf{B}}^{\mathrm{T}}(w_k)\boldsymbol{\beta}]H(t-\hat{t}_k)\right\} \tag{66}$$

and the interfacial stress as

$$\mathbf{t}_2(x_1,t) = -\frac{1}{\pi}\sum_{k=4}^{6}\mathrm{Im}\left\{\frac{1}{w_k}\frac{\partial w_k}{\partial x_1}\widehat{\mathbf{M}}_2(w_k)\mathbf{M}^{-1}(w_k)[\widehat{\mathbf{A}}^{\mathrm{T}}(w_k)]^{-1}\mathbf{I}_{k-3}[\widehat{\mathbf{A}}^{\mathrm{T}}(w_k)\mathbf{F} + \widehat{\mathbf{B}}^{\mathrm{T}}(w_k)\boldsymbol{\beta}]H(t-\hat{t}_k)\right\}. \tag{67}$$

In summary, the particle velocity due to the line force and the dislocation in material 1 may be expressed as

$$\dot{\mathbf{u}}(x_1,x_2,x_t) = \mathbf{G}_f(x_1,x_2,t)\mathbf{F} + \mathbf{G}_b(x_1,x_2,t)\boldsymbol{\beta}, \tag{68}$$

where

$$\mathbf{G}_f = \mathbf{G}_f^{(0)} + \mathbf{G}_f^{(1)} \tag{69}$$

with $\mathbf{G}_f^{(0)}$ and $\mathbf{G}_f^{(1)}$ given by

$$\mathbf{G}_f^{(0)}(x_1,x_2,t) = -\frac{1}{\pi}\,\mathrm{Im}\left\{\sum_{k=1}^{3}\frac{\partial w_k}{\partial x_1}\mathbf{a}_k(w_k)\mathbf{a}_k^{\mathrm{T}}(w_k)H(t-\hat{t}_k)\right\}, \tag{70}$$

$$\mathbf{G}_f^{(1)}(x_1,x_2,t) = \frac{1}{\pi}\sum_{k=4}^{6}\sum_{j=1}^{3}\mathrm{Im}\left\{R_{kj}(w_{kj})\frac{\partial w_{kj}}{\partial x_1}\mathbf{a}_j(w_{kj})\mathbf{a}_k^{\mathrm{T}}(w_{kj})H(t-\hat{t}_{kj})\right\} \tag{71}$$

and

$$\mathbf{G}_b = \mathbf{G}_b^{(0)} + \mathbf{G}_b^{(1)} \tag{72}$$

with $\mathbf{G}_b^{(0)}$ and $\mathbf{G}_b^{(1)}$ given by

$$\mathbf{G}_b^{(0)}(x_1,x_2,t) = -\frac{1}{\pi}\,\mathrm{Im}\left\{\sum_{k=1}^{3}\frac{\partial w_k}{\partial x_1}\mathbf{a}_k(w_k)\mathbf{b}_k^{\mathrm{T}}(w_k)H(t-\hat{t}_k)\right\}, \tag{73}$$

$$\mathbf{G}_b^{(1)}(x_1,x_2,t) = \frac{1}{\pi}\sum_{k=4}^{6}\sum_{j=1}^{3}\mathrm{Im}\left\{R_{kj}(w_{kj})\frac{\partial w_{kj}}{\partial x_1}\mathbf{a}_j(w_{kj})\mathbf{b}_k^{\mathrm{T}}(w_{kj})H(t-\hat{t}_{kj})\right\}, \tag{74}$$

The particle velocity due to the line force and the dislocation in material 2 is

$$\dot{\mathbf{u}}^*(x_1,x_2,t) = \mathbf{G}_f^*(x_1,x_2,t)\mathbf{F} + \mathbf{G}_b^*(x_1,x_2,t)\boldsymbol{\beta}, \tag{75}$$

where $\mathbf{G}_f^*$ and $\mathbf{G}_b^*$ given by

$$\mathbf{G}_f^*(x_1,x_2,t) = \frac{1}{\pi}\sum_{k=4}^{6}\sum_{j=4}^{6}\mathrm{Im}\left\{T_{kj}(w_{kj}^*)\frac{\partial w_{kj}^*}{\partial x_1}\mathbf{a}_j^*(w_{kj})\mathbf{a}_k^{\mathrm{T}}(w_{kj}^*)H(t-\hat{t}_{kj}^*)\right\}, \tag{76}$$

$$\mathbf{G}_b^*(x_1,x_2,t) = \frac{1}{\pi}\sum_{k=4}^{6}\sum_{j=4}^{6}\mathrm{Im}\left\{T_{kj}(w_{kj}^*)\frac{\partial w_{kj}^*}{\partial x_1}\mathbf{a}_j^*(w_{kj}^*)\mathbf{b}_k^{\mathrm{T}}(w_{kj}^*)H(t-\hat{t}_{kj}^*)\right\}. \tag{77}$$

Since the velocity field due to $\mathbf{F}H(t)$ is the same as the displacement field due to $\mathbf{F}\delta(t)$. $\delta(\,)$ being the Dirac delta function, $\mathbf{G}_f$ and $\mathbf{G}_f^*$ may also be regarded as the Green's function for an impulsive force. Similarly $\mathbf{G}_b$ and $\mathbf{G}_b^*$ are the Green's function due to an impulsive dislocation.

4. Numerical examples

The results derived in Section 3 were calculated for an isotropic and an anisotropic materials in this section. A numerical scheme for computing the relevant quantities is outlined in the Appendix.

Numerical calculations were first made for the case of an isotropic bimaterial treated by Ma and Huang (1996). In their paper, the applied point force was assumed in the x_2 direction with the magnitude $F = 2\sigma_0$ and located at $(0, h)$. The slownesses were taken as $a_1{:}b_1{:}a_2{:}b_2 = 1/5{:}1/4{:}1/3{:}1/2$, where a_i and b_i be the slowness of the longitudinal wave and the transverse wave for the material i, respectively, and the ratio of the shear moduli of the two materials were assumed as $\mu_2/\mu_1 = 0.3$. The variations of the dimensionless stress $h\sigma_{yy}/\sigma_0$

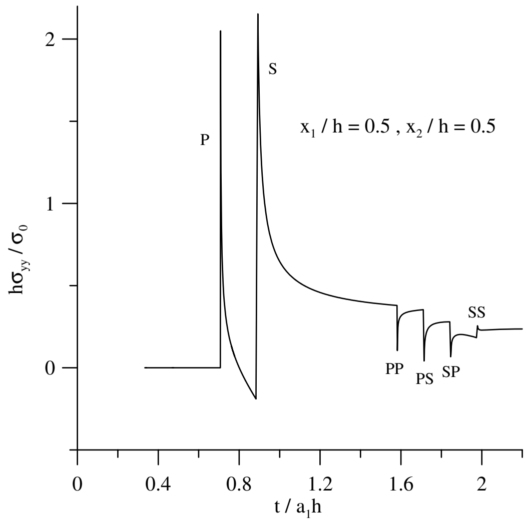

Fig. 2. $h\sigma_{yy}/\sigma_0$ for the isotropic material at $x_1/h = 0.5$ and $x_2/h = 0.5$.

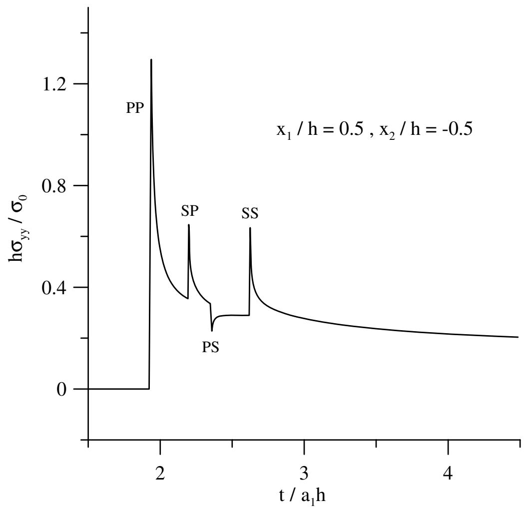

Fig. 3. $h\sigma_{yy}/\sigma_0$ for the isotropic material at $x_1/h = 0.5$ and $x_2/h = -0.5$.

with the dimensionless time t/a_1h at $x_1/h = 0.5$ and $x_2/h = 0.5$ were computed using Eq. (36). The result is shown in Fig. 2. Fig. 3 shows the variations at $x_1/h = 0.5$ and $x_2/h = -0.5$ calculated by Eq. (40). In both figures the arrivals of two bulk waves are denoted by P and S, and the arrivals of four reflected or refracted waves are denoted by PP, PS, SP and SS where the first letter represents the incident wave and the second letter indicates the reflected or refracted wave. The present results are in excellent agreement with those in Ma and Huang (1996).

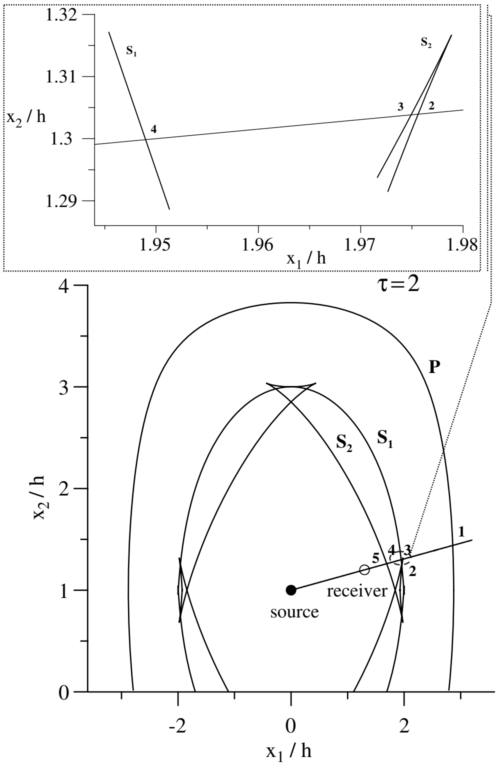

Fig. 4. The transient body wavefronts of the bimaterial – GaAs at $\tau = 2$.

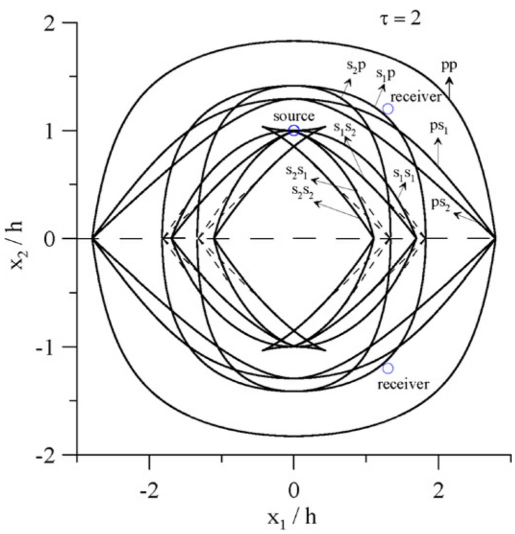

Fig. 5. The transient reflected and transmitted wavefronts for GaAs at $\tau = 2$.

The second example is concerned with a GaAs bimaterial. GaAs crystal is of cubic symmetry. The elastic constants of GaAs with respect to the symmetry axes in units of 100 GPa are $C_{11} = 1.19$, $C_{12} = 0.538$, and $C_{44} = 0.595$ (Bateman et al., 1959). The bimaterial is formed by cutting an infinite body of GaAs into two half-spaces along the symmetry (x_1, x_3) plane, rotating the half-space $x_2 \geqslant 0$ by 10° and the half-space $x_2 \leqslant 0$ by $-10°$ about the x_2 axis, and re-welding the two rotated half-spaces. The source was assumed at $(0, h)$ and the receivers at $(1.3h, 1.2h)$ and $(1.3h, -1.2h)$.

The Green's functions given by Eqs. (69), (72), (76), and (77) were computed and expressed in the following dimensionless form:

$$\mathbf{G}_f(x_1, x_2, t) = \frac{1}{\pi\rho c_0 h}\overline{\mathbf{G}}_f(\bar{x_1}, \bar{x_2}, \tau), \quad \mathbf{G}_b(x_1, x_2, t) = \frac{c_0}{\pi h}\overline{\mathbf{G}}_b(\bar{x_1}, \bar{x_2}, \tau),$$
$$\mathbf{G}^*_f(x_1, x_2, t) = \frac{1}{\pi\rho c_0 h}\overline{\mathbf{G}}^*_f(\bar{x_1}, \bar{x_2}, \tau), \quad \mathbf{G}^*_b(x_1, x_2, t) = \frac{c_0}{\pi h}\overline{\mathbf{G}}^*_b(\bar{x_1}, \bar{x_2}, \tau),$$

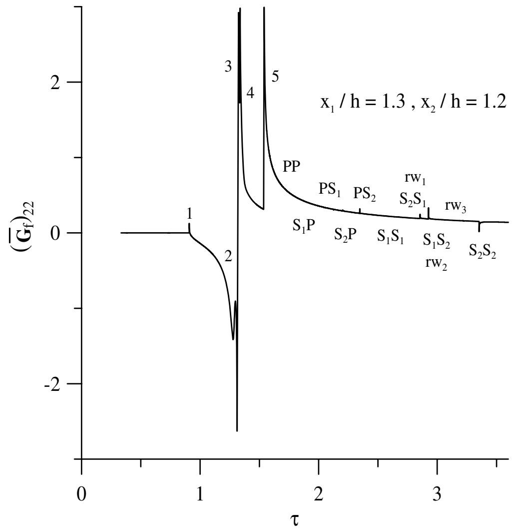

Fig. 6. $(\overline{\mathbf{G}}_f)_{22}$ for GaAs at $x_1/h = 1.3$ and $x_2/h = 1.2$.

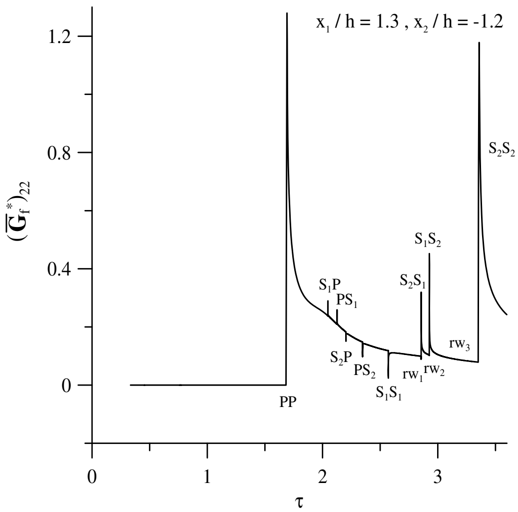

Fig. 7. $(\overline{\mathbf{G}}^*_f)_{22}$ for GaAs at $x_1/h = 1.3$ and $x_2/h = -1.2$.

where $\bar{x}_i = x_i/h,\ i = 1, 2,\ \tau = tc_0/h$, and $c_0 = \sqrt{C_{44}/\rho}$. Fig. 4 shows the direct wavefronts of the GaAs at $\tau = 2$. In Fig. 4, all body waves have passed through the receiver at $(1.3h, 1.2h)$ at the selected time. Fig. 5 displays the reflected and transmitted wavefronts at $\tau = 2$. Figs. 6 and 7 give the components $(\overline{\mathbf{G}}_f)_{22}$ and $(\overline{\mathbf{G}}_f^*)_{22}$, respectively. The components $(\overline{\mathbf{G}}_b)_{11}$ and $(\overline{\mathbf{G}}_b^*)_{11}$, respectively, are shown in Figs. 8 and 9. The arrival times of bulk waves, reflected waves and transmitted waves are labeled in all figures.

Figs. 6–9 clearly indicate that, in addition to the direct waves and reflected or transmitted waves, there are also refracted waves present. The condition for the occurrence of a refracted ray at (x_1, x_2) is (Wu, 2001)

$$x_1 > h \tan \phi_i + |x_2| \tan \phi_r. \tag{78}$$

Here ϕ_i and ϕ_r denote the incident angle from the source to the interface and the refracted angle from the interface to the receiver, respectively. When a ray of S_1 or S_2 satisfying Eq. (78) reaches the interface of the bimaterial, it first travels as P or S_1 wave along the interface, and then propagates as S_1 or S_2 wave from the interface to the receiver. In Figs. 6–9, the symbol rw_1 represents the refracted wave S_2PS_1, rw_2 for S_1PS_2 and rw_3 for S_2PS_2.

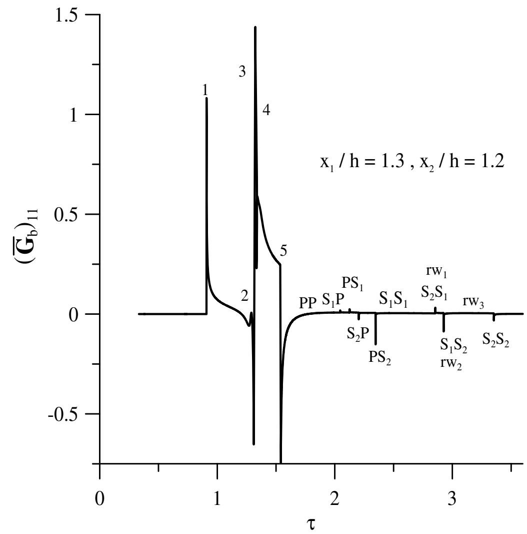

Fig. 8. $(\overline{\mathbf{G}}_b)_{11}$ for GaAs at $x_1/h = 1.3$ and $x_2/h = 1.2$.

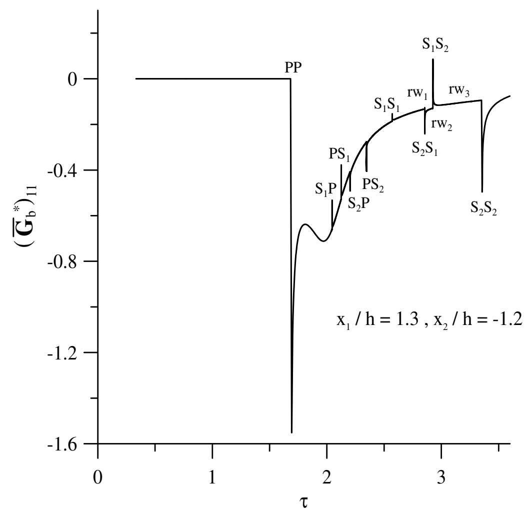

Fig. 9. $(\overline{\mathbf{G}}_b^*)_{11}$ for GaAs at $x_1/h = 1.3$ and $x_2/h = -1.2$.

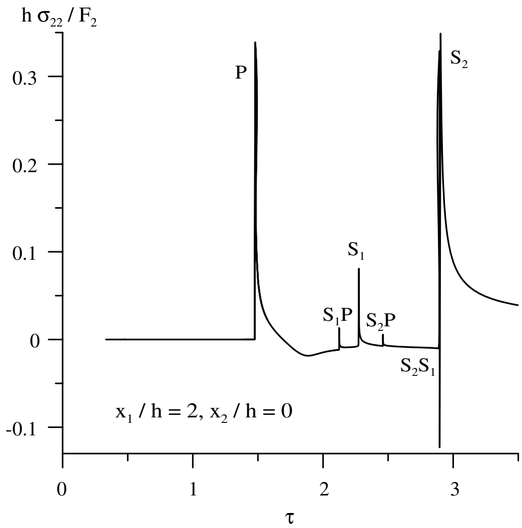

Fig. 10. $h\sigma_{22}/F_2$ for GaAs at $x_1/h = 2$ and $x_2/h = 0$.

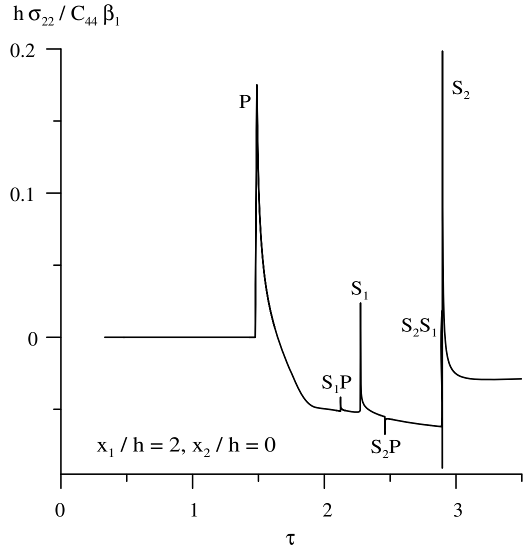

Fig. 11. $h\sigma_{22}/C_{44}\beta_1$ for GaAs at $x_1/h = 2$ and $x_2/h = 0$.

Eq. (67) was also computed for the time history of the normal stress at $x_1/h = 2$ and $x_2/h = 0$ on the interface. Fig. 10 gives the normal stress $h\sigma_{22}/F_2$ due to a vertical line force F_2 and Fig. 11 shows the normal stress $h\sigma_{22}/C_{44}\beta_1$ due to an edge dislocation of Burgers vector β_1. In Figs. 10 and 11, the symbol S_1P denotes the P wave refracted from the S_1 wave. The other two refracted waves are similarly denoted as S_2 P and S_2S_1. The arrival time for the refracted wave S_2S_1 is very close to that for the S_2 wave.

5. Conclusion

In this study transient Green functions are obtained for a general anisotropic bimaterial due to an interior dynamic line force or dislocation. The Green functions are derived using a novel formulation developed by Wu and Chen (2006), which does not require integral transforms. It is shown that in the limit as $t \to \infty$, the static solution given by Ting (1996) is recovered. It is also shown that when the dynamic sources are located at the interface, Green functions reduce to those reported by Wu (2003). Numerical examples are given to illustrate the effects due to reflection and transmission by the interface. The analysis presented here serves as a basis for studying dynamic sources in anisotropic elastic multi-layered media.

Acknowledgements

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Appendix A

In this appendix a numerical scheme for calculating w_k, w_{kj}, and w^*_{kj} for given x_1, x_2 and t from Eqs. (33), (37) and (41), respectively, and the related quantities is described.

Eq. (33), (37) and (41) are in the following form

$$\phi(w) = wt - x_1 - \tilde{p}(w)x_2 + p_k(w)h = 0, \quad k = 4,5,6, \tag{A.1}$$

where $\tilde{p}(w) = p_k(w)$ for $w_k, \tilde{p}(w) = p_j(w),\ j = 1,2,3$, for w_{kj}, and $\tilde{p}(w) = p^*_j(w),\ j = 4,5,6$, for w^*_{kj}. The functions $p_j(w)$ are the eigenvalues of $\mathbf{N}(w)$ of Eq. (20) with the elastic constants of material 1 and $p^*_j(w)$ are the eigenvalues of $\mathbf{N}^*(w)$ with the elastic constants of material 2. An iteration process for solving Eq. (A.1) is as follows. Take $w^{(0)}$ as an initial trial value for w and expand $\phi(w)$ about $w^{(0)}$ up to the second order term in Taylor's series,

$$\phi(w) \approx \phi^{(0)} + \left(\frac{\partial \phi}{\partial w}\right)^{(0)} \Delta w^{(0)} + \frac{1}{2}\left(\frac{\partial^2 \phi}{\partial w^2}\right)^{(0)} (\Delta w^{(0)})^2, \tag{A.2}$$

where $\Delta w^{(0)} = w - w^{(0)}$, and $(f)^{(0)} = f(w^{(0)})$. An approximate solution of $\Delta w^{(0)}$ can be obtained by substituting Eq. (A.2) into Eq. (A.1) as

$$\Delta w^{(0)} = (-b + \sqrt{b^2 - ac})/a, \tag{A.3}$$

where a, b, and c are given by

$$a = -\tilde{p}''(w^{(0)})x_2 + p''_k(w^{(0)})h, \quad b = t - \tilde{p}'(w^{(0)})x_2 + p'_k(w^{(0)})h, \quad c = 2\phi^{(0)}.$$

Let $w^{(1)} = w^{(0)} + \Delta w^{(0)}$. If $|\phi(w^{(1)})| < \varepsilon$, where ε is a preset error, then $w^{(1)}$ is accepted as the solution. Otherwise Eq. (A.3) is used again with $w^{(0)}$ replaced by $w^{(1)}$. The process is repeated until the error criterion is met.

In our calculations, the numerical procedure was started with a sufficiently large time for which accurate approximations given by Eq. (58) for w_{kj} and w^*_{kj} are available. The time was subsequently decreased and the values of w_{kj} and w_{kj} at the previous time step were used as the initial trial values. Our experience is that it rarely takes more than ten iterations to achieve a numerical accuracy of 10^{-5}.

In evaluating Eq. (A.3) $p_s(w)$, $s = 1$–6, or $p^*_j(w),\ j = 4,5,6$ and their first and second derivatives for a given w are required. A procedure for computing these quantities are given here for $p_s(w)$ associated with material 1. Those for $p^*_j(w)$ can be obtained by simply using the elastic constants of material 2. The values of p_s are obtained simply by substituting w into Eq. (20) and compute the eigenvalues of $\mathbf{N}(w)$. The corresponding vectors $\mathbf{a}_s(w)$ and $\mathbf{b}_s(w)$ are also obtained. To find the first derivative p'_s, differentiate Eq. (10) with respect to w and setting $p = p_s$ and $\mathbf{a} = \mathbf{a}_s$. The result is

$$\mathbf{D}'(p_s, w)\mathbf{a}_s + \mathbf{D}(p_s, w)\mathbf{a}'_s = \mathbf{0}, \tag{A.4}$$

where $\mathbf{D}' = p'_s(\mathbf{S} + \mathbf{S}^{\mathrm{T}} + 2p_s\mathbf{W}) - 2\rho w\mathbf{I}$. Pre-multiplying Eq. (A.4) by $\mathbf{a}^{\mathrm{T}}_s$ and using Eq. (10) leads to

$$\mathbf{a}^{\mathrm{T}}_s\mathbf{D}'(p_s, w)\mathbf{a}_s = 0 \tag{A.5}$$

from which p'_s is obtained as

$$p'_s = 2\rho w\mathbf{a}^{\mathrm{T}}_s\mathbf{a}_s, \tag{A.6}$$

where Eqs. (19) and (25) have been used. Eq. (A.6) contains only the known vector $\mathbf{a}_s(w)$. Note that $p_j, j = 1, 2, 3$, are selected such that the imaginary parts are positive if they are complex or $p'_j > 0$ when they are real. The latter condition can be checked using Eq. (A.6). Similarly $p_k, k = 4, 5, 6$, are selected such that the imaginary parts are negative if they are complex or $p'_k < 0$ when they are real. To determine the second derivative p''_s, differentiate Eq. (10) twice with respect to w and setting $p = p_s$ and $\mathbf{a} = \mathbf{a}_s$. The result is

$$\mathbf{D}''(p_s, w)\mathbf{a}_s + 2\mathbf{D}'(p_s, w)\mathbf{a}'_s + \mathbf{D}(p_s, w)\mathbf{a}''_s = \mathbf{0}, \tag{A.7}$$

where $\mathbf{D}'' = p''_s(\mathbf{S} + \mathbf{S}^{\mathrm{T}} + 2p_s\mathbf{W}) + 2(p'_s)^2\mathbf{W} - 2\rho\mathbf{I}$. Pre-multiplying Eq. (A.7) by $\mathbf{a}_s^{\mathrm{T}}$ and using Eq. (10) yields

$$\mathbf{a}_s^{\mathrm{T}}\mathbf{D}''(p_s, w^{(0)})\mathbf{a}_s + 2\mathbf{a}_s^{\mathrm{T}}\mathbf{D}'(p_s, w^{(0)})\mathbf{a}'_s = 0. \tag{A.8}$$

From Eq. (A.8), p''_s may be expressed as

$$p''_s = 2\left[\rho\mathbf{a}_s^{\mathrm{T}}\mathbf{a}_s - (p'_s)^2\mathbf{a}_s^{\mathrm{T}}\mathbf{W}\mathbf{a}_s + (\mathbf{a}'_s)^{\mathrm{T}}\mathbf{D}\mathbf{a}'_s\right], \tag{A.9}$$

where Eqs. (19), (25) and (A.4) have been used. The terms on the right side of Eq. (A.9) are known except $\mathbf{a}'_s$. The vector $\mathbf{a}'_s$ can be calculated from Eq. (A.4) as follows. Let D_k and $\mathbf{d}_k$, respectively, be the eigenvalues and the corresponding eigenvectors of $\mathbf{D}(p_s, w)$, i.e.,

$$\mathbf{D}(p_s, w)\mathbf{d}_k = D_k\mathbf{d}_k. \tag{A.10}$$

Since $\mathbf{D}(p_s, W)$ is symmetric, the eigenvectors can be normalized such that $\mathbf{d}_j^{\mathrm{T}}\mathbf{d}_k = \delta_{jk}$. Let $\mathbf{a}'_s$ be expressed as

$$\mathbf{a}'_s = \sum_{j=1}^{3} g_k\mathbf{d}_k. \tag{A.11}$$

As $|\mathbf{D}(p_s, w)| = 0$ and at least one of D_k is zero. Substitution of Eq. (A.11) into Eq. (A.4) gives

$$\sum_{k=1}^{n} g_k D_k\mathbf{d}_k = -\mathbf{D}'(p_s, w^{(0)})\mathbf{a}_s, \tag{A.12}$$

where $n < 3$ is the number of nonzero D_k. From Eq. (A.12),

$$g_k = -\frac{1}{D_k}\mathbf{d}_k^{\mathrm{T}}\mathbf{D}'(p_s, w^{(0)})\mathbf{a}_s, k = 1 \text{ to } n \tag{A.13}$$

with Eqs. (A.11) and (A.13) the tern $(\mathbf{a}'_s)^{\mathrm{T}}\mathbf{D}\mathbf{a}'_s$ in Eq. (A.8) is given by

$$(\mathbf{a}'_s)^{\mathrm{T}}\mathbf{D}\mathbf{a}'_s = \sum_{k=1}^{n} g_k^2 D_k. \tag{A.14}$$

As the iteration process is completed for w_{kj}, $p_s(w_{kj})$, $p'_s(w_{kj})$, $\mathbf{a}_s(w_{kj})$ and $\mathbf{b}_s(w_{kj})$, $s = 1$ to 6, are also obtained. The derivatives $p'_s(w_{kj})$ are used to calculate

$$\partial w_{kj}/\partial x_1 = 1/\left(t - p'_j(w_{kj})x_2 + p'_k(w_{kj})h\right).$$

The vectors $\mathbf{a}_s(w_{kj})$ and $\mathbf{b}_s(w_{kj})$, $s = 1, 2, 3$, are used to construct the matrices $\mathbf{A}(w_{kj})$ and $\mathbf{B}(w_{kj})$, respectively, defined by Eqs. (46) and (47). The matrices $\widehat{\mathbf{A}}^*(w_{kj})$ and $\widehat{\mathbf{B}}^*(w_{kj})$ defined by Eqs. (50) and (51), respectively, are obtained by substituting w_{kj} into Eq. (20) with the material constants of material 2 and calculating the eigenvectors $\boldsymbol{\xi}_j^*$, $j = 4, 5, 6$. The four matrices $\mathbf{A}(w_{kj})$, $\mathbf{B}(w_{kj})$, $\widehat{\mathbf{A}}^*(w_{kj})$ and $\widehat{\mathbf{B}}^*(w_{kj})$ are needed for $R_{kj}(w_{kj})$ of Eq. (57).

At the completion of the iteration process for w_{kj}^*, not only $p_s(w_{kj}^*)$, $p'_s(w_{kj}^*)$, $\mathbf{a}_s(w_{kj}^*)$ and $\mathbf{b}_j(w_{kj}^*)$, $s = 1$ to 6, but also $p_j^*(w_{kj}^*)$, $p_j^{*\prime}(w_{kj}^*)$, $\mathbf{a}_j^*(w_{kj}^*)$ and $\mathbf{b}_j^*(w_{kj}^*)$, $j = 4, 5, 6$ are obtained. The derivatives $p'_k(w_{kj}^*)$ and $p_j^{*\prime}(w_{kj}^*)$, $j, k = 4, 5, 6$ are used to calculate

$$\partial w_{kj}^*/\partial x_1 = 1/\left(t - p_j^{*\prime}(w_{kj}^*)x_2 + p'_k(w_{kj}^*)h\right).$$

The vectors $\mathbf{a}_s(w^*_{kj})$ and $\mathbf{b}_s(w^*_{kj}),\ s = 1, 2, 3$, are used to construct the matrices $\mathbf{A}(w^*_{kj})$ and $\mathbf{B}(w^*_{kj})$. The vectors $\mathbf{a}^*_j(w^*_{kj})$ and $\mathbf{b}^*_j(w^*_{kj}),\ j = 4, 5, 6$, are used to establish $\widehat{\mathbf{A}}^*(w^*_{kj})$ and $\widehat{\mathbf{B}}^*(w^*_{kj})$. The four matrices $\mathbf{A}(w^*_{kj}),\ \mathbf{B}(w^*_{kj}),\ \widehat{\mathbf{A}}^*(w^*_{kj})$ and $\widehat{\mathbf{B}}^*(w^*_{kj})$ are required for $T_{kj}(w^*_{kj})$ of Eq. (57).

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